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WIND-TUNNEL INVESTIGATIONS

ON FLEXURAL-TORSIONAL WING FLUTTER

By H. Voigt

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For the purpose of testing the theory of an oscillating airfoil of two degrees of freedom, a wing was mounted in the wind tunnel between two walls in such a manner that it could execute vertical (flexural) oscillations as well as torsional oscillations about an arbitrary axis parallel to the span. It was possible to vary the inertia and elasticity parameters and also to increase artificially the negligibly small natural damping of the system. The oscillations were recorded to a strongly magnified scale. The experimentally determined critical (or flutter) velocities fully agree with the theoretical ones of Wagner and Küssner within the limits of computational and measuring accuracy. An extremely narrow wing without end walls (three-dimensional problem) showed the same oscillations as one with end walls (two-dimensional problem).

I. OBJECT OF THE TESTS

In Germany in the past decade, methods have been presented by Blenk and Liebers (reference 1), by Küssner (references 2 and 8), and by Wagner and Kassner (references 5 to 7), for computing the critical velocity at which unstable wing oscillations may be expected (flutter velocity). Outside of Germany the work of Theodorsen (reference 3) deserves mention. Blenk and Liebers neglect the reaction of the vortices leaving the oscillating airfoil and hence give only useful approximations for oscillations with low reduced frequency $\omega$ or high reduced velocity $V$. The expressions of Küssner, Wagner and Kassner, and Theodorsen, although derived from various initial assumptions, lead to the same solution determinants and give the same results. In the present paper the theoretical computations are car-

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ried out partly according to Küssner, but more largely according to the graphical method of Wagner and Kassner.

The present tests are intended to verify the correctness of the theory for the two degrees of freedom of wing deflection and wing rotation.  For this purpose, it was necessary to mount as light a wing as possible in such a manner that the natural damping of the system was a minimum, and so as to permit the parameters to be quickly and reliably varied.  By far the greatest time expenditure was required for the accurate determination of the system parameters.  By recording the oscillations, an analysis of the phase and amplitude relations and an exact determination of the frequency were made possible.

The tests extended over a period of 2-1/2 years.  The results were mostly communicated to interested parties (Henschel Airplane Company, Technical Flight Institute at Berlin, Institute for Aerodynamics at Göttingen, Junkers, and Heinkel) and were conducted in close cooperation with the first two named for whose encouragement and aid appreciation is hereby expressed.

The present report includes only comparison with the theory within the scope mentioned.  Results of first tests with ailerons, the relations between steady and unsteady oscillations (flutter), the effect of externally induced vibrations, and the effect of vibration dampers are dealt with in reference 4.

## II. COLLECTION OF SYMBOLS USED

$c$, flexural stiffness of wing

$\epsilon\, t$, position of elastic axis behind center of pressure

$\eta\, t$, elastic radius $= \sqrt{\dfrac{M/\varphi}{c}}$

$i\, t$, radius of gyration of wing with respect to center of gravity

$\mu$, reduced wing mass or mass ratio $= \dfrac{m_F}{m_L}$

$m_F$, mass of wing with end attachment parts, oscillating springs, additional weights, and recording apparatus

$m_L$,     mass of the air cylinder described by the wing chord

$M/\varphi$,     torsional stiffness of the wing

$v$,     frequency of the oscillations

$\sigma t$,     position of the center of gravity behind the elastic axis

$t$,     wing chord

$v$,     wind velocity

$v_k$,     critical or flutter velocity

$V$,     reduced velocity

$\omega$,     reduced frequency

$\varphi$,     phase between flexural and torsional oscillations

## III. DESCRIPTION OF THE TEST SET-UP

The results, presented in sections VI and VII, were obtained on a wing of 250 mm chord, 800 mm span, and 30 mm thickness. The wing had an NACA profile 0012/63, was constructed of wood, its forward portion covered by plywood and after portion by Japan tissue, and could be considered as stiff.

The wing was suspended between the two end walls, the suspension consisting of two end parts attached to the tips of the wing and to each of which was fixed four helical springs adjusted to the same stiffness. To obtain the drag a drag bracing member was employed, connected at each end to a pin at 32 percent chord. The position of the elastic axis and the elastic radius was determined by the points of attachment of the vertical springs and their distances apart. For varying the mass, the position of the center of gravity and the radius of gyration weights were employed, which were clamped to the end attachment parts (figs. 1, 2, and 3).

The oscillations were recorded by two concave mirrors, one of which recorded the rotation (or torsion) of the wing

and the second, by means of a simple lever transmission, the deflection, i.e., the vertical motion, the record being made on light-sensitive paper (fig. 2). This method has the advantage that the rotation need not necessarily be measured from two deflection amplitudes but can be recorded at the same position of the film with the deflection of an arbitrary point, for example, the elastic axis. It is possible, furthermore, to choose the recording scale arbitrarily large. The scale was determined by calibration, 1 mm on the record corresponding to 0.157 mm of wind deflection and 0.1315° rotation.

The natural damping of the set-up was negligibly small as is shown by the oscillation curve (fig. 4). It was possible to increase the damping artificially by permitting the lower springs, which were situated in vessels (fig. 3), to operate in glycerine-water mixtures of various composition. The largest investigated damping decrement $\vartheta = 0.4$ lies above all value of the material damping decrements of the wing that occur in practice.

## IV. DETERMINATION OF THE PARAMETERS

The vibrating mass of the system consists of the mass of the wing, the side attachments, the weights and the suspension with drag bracing and mirror mounting. The first-named components were determined by weighing, the latter by frequency determinations in oscillation tests. The associated oscillating air mass was determined by oscillation computations.

The position of the center of gravity of the wing with the side-attachment parts was determined by weighing of the wing suspended on two V wires. The center of gravity of the total system was computed.

The elastic data, that is, flexural stiffness, torsional stiffness, and elastic axis, may readily be determined theoretically from the stiffness of the individual springs and their positions of attachment. The relations were affected, however, by the drag bracing, the finite, vertical distance apart of the spring-attachment points at the sides, the finite length of the spring extension, and, to a slight extent, by the lever of the deflection mirror; so that for the accurate determination of the elastic data, stiffness measurements with various spring distances and elastic axes were required.

The accurate determination of the moments of inertia was difficult, since the particularly light wing was affected during its oscillation by the mass of air vibrating with it. The most accurate method appeared to be by means of the oscillation tests of the wings in the test set-up, the wing being fixed at its axis through the center of gravity so that it could perform only torsional vibrations, and the paper covering cut out and attached rolled up.

The damping was likewise determined by oscillation tests. In order to exclude the quite considerable air damping, whose decrement $\vartheta/\pi$ was determined up to 0.03, the wing was replaced by a tube which connected the two side attachments. The system was periodically excited by rubber tension. The rubber tension was removed by burning a thread when the system in its flexural or torsional oscillations vibrated in resonance. In this way, natural vibrations without secondary vibrations were obtained. This method was also applied in the determination of the moments of inertia.

## V. TEST PROCEDURE,

## ANALYSIS OF SEVERAL OF THE OSCILLATION RECORDS

The wind velocity was first gradually increased in the wind tunnel until flutter was set up. This was generally uniform so that the oscillation was of constant amplitude as shown in figure 5a. To each wind velocity there corresponded a definite amplitude (fig. 6). In contrast to the theory, which is based on the assumption of infinitely small amplitudes, a damping by the air thus occurred at finite amplitudes. The sharp theoretical separation line between the unstable and damped oscillations becomes a broad band of the stable region and the upper limit of the critical velocity is determined by the strength of the airplane wing. In the diagrams, figures 8 to 13, velocities at which uniform amplitudes of about two percent of the chord occur are denoted by the sign =.

The wind velocity was then slowly reduced and the point (>) noted at which the oscillations stopped of themselves. This point must correspond most accurately to the theory, since here the limiting case of infinitely small oscillations with vortex field extending very far behind was most nearly satisfied.

In particular cases, especially for small distance of
the center of gravity behind the elastic axis and at high
dynamic pressures, it was not possible to obtain stable
oscillations; oscillations were more often set up suddenly,
the wing striking against the stops and being brought to
rest again.  These cases, of which a mild example is plot-
ted in figure 5b are denoted in the diagrams by the sign
< >.  The irregularity is explained by the fact that the
air-stream disturbances originating from the blower, which
could not be entirely removed by the honeycomb at some par-
ticular instants, predominated over the action of the air
forces.  In still more extreme cases, the oscillations on
account of their irregularity could hardly be considered as
flutter oscillations (denoted by  u).

Figure 5c shows, finally, a wing at the limit of stat-
ic stability at which limit small changes in the angle of
attack lead to an upward or downward tipping of the wing.

Figure 7 shows oscillation records with the test ap-
paratus in its first form in which by dry friction damping,
whose decrement decreases with increasing amplitude, an un-
stabilizing value was obtained, resulting in a particularly
unstable oscillation.  Frequencies, amplitudes, and phase
angles between rotation and bending are given on figures 5a
and 7.  Further analysis of the curves and a detailed dis-
cussion of the effect of the phase angle and the effect of
external excitation on the phase angle are found in refer-
ence 4.

VI.  COMPARISON WITH THE THEORY FOR THE CASE WITHOUT DAMPING

In order to test the correctness of the theory as com-
prehensively as possible not only individual points but also
entire test series were compared with the theory.  In figure
8 the position of the center of gravity, and hence also the
radius of gyration, was varied.  This results in only a slight
change in the critical or flutter velocity.  At the center
of the curves there is a slight scattering of the computed
and test points.  For the computation the graphical method of
Kassner was employed.  In the tests a wire of the spring sus-
pension appears to have rubbed somewhere.  The test results
are extremely sensitive to even the smallest disturbances of
this kind.

In figures 9, 10, and 11, the elastic radius is the

principal variable. Variation of the latter also produces
a slight variation in the radius of gyration. The three
series of tests differ in the various values of the radius
of gyration, the added weights being of equal size and with
equal position of the center of gravity but with various
distances from the center of gravity of the entire system.
In these series of tests the critical velocity is strongly
affected by a change in the elastic radius. In spite of
the extraordinary sensitivity of the measurements, it is
possible to speak of sufficient agreement between theory
and experiment. A fact to be noted is that in the measure-
ments flutter vibrations could be observed up to the neigh-
borhood of zero velocity.

In figure 12 the results of three series of tests are
simultaneously presented for which only the stiffness of the
springs was varied. This results also in a slight change
in the radius of gyration and the oscillating mass. The in-
dependent variable in this series of tests is the position
of the center of gravity. In the three tests, there were
equal reduced velocities V, that is, equal oscillation
conditions for different wind velocities. Since, in each
case there is agreement between theory and experiment, it
is thus shown that a change in the Reynolds Number has no
effect on the results. At the small Reynolds Numbers em-
ployed, the "angle-of-attack sensitivity" $d c_a/d \alpha$ of the
stationary flow is 25 percent lower than in the normal
flight range as was established, in agreement with similar
measurements, by investigation of the wing itself. Figure
12 shows, as also the general agreement between theory and
experiment in the previous figures, that during the oscil-
lation all boundary layer processes are to a large extent
eliminated and that it is correct to use in the computation
the theoretical angle of attack sensitivity (about $2 \pi$).

In order to test also very light wings with the least
mass ratios ($\mu = 2.6$), a wing of double the chord and equal
in thickness ratio was tested on the same test set-up. It
was here not possible to obtain complete agreement between
theory and experiment. On account of the turbulence of the
jet, that is, the irregularity of the flow coming from the
blower or guide vanes and which was not entirely eliminated
by the honeycomb, the disturbance of the oscillatin airfoil
was greater the smaller its mass. In the case of very light
wings, the critical velocity can thus be considerably in-
creased by these distrubances. It is also shown that the
jet of a wind tunnel has a large tendency to oscillate.

Through the use of a wing chord of 500 mm, the jet becomes
highly loaded. It is therefore conceivable that to the
oscillations of the wing there are added jet oscillations
of some kind which may displace the critical velocity con-
siderably below or above. It is therefore planned to car-
ry out several supplementary tests in the larger wind tun-
nel on light wings and to provide special honeycombs for
quieting the jet. From these tests, it may be concluded
that similarly in flight through strongly turbulent air
flexural-torsional flutter oscillations of particularly
light airfoils may be delayed.

## VII. COMPARISON WITH THE THEORY

## IN THE CASE OF VARIABLE DAMPING

On figure 13 the critical velocity has been plotted
against the variable damping decrement. The test data are
taken from figure 8. Theory and experiment show satisfac-
tory agreement so that also for the damping the theoretical
expression should be considered as correct.

On figure 14, finally are plotted several results
that have been obtained with the first test set-up in 1935.
The natural damping was here still rather large and corre-
sponded approximately to the material damping of the wing.
The damping was determined by vibration tests and includes
the air damping of the oscillating airfoil. Here, too,
there is good agreement between theory and experiment.

## VIII. TESTS FOR THE THREE-DIMENSIONAL PROBLEM

The starting point was the following consideration.
On a wing element which possesses a very small span in
comparison with its chord, there is first investigated the
two-dimensional oscillating condition by placing the wing
between two end walls. If these end walls are now removed,
the wing element is exposed to a more intense flow at its
tips than a wing element as part of an airfoil with normal
aspect ratio. All the effects of the tip flow must there-
fore in this model appear particularly emphasized. Con-
versely, an agreement of the critical velocity in this lim-
iting case would indicate that the effect of the tip flow
of the wing does not show up in the wing flutter.

The model employed had the following data (see fig. 15):

Chord     500 mm        Aspect ratio        0.6

Span      300 mm        Flexural elasticity   0.68 kg/cm

The wing was suspended on four springs, which were situated outside the air stream, four simple V wires leading from the springs to the wing. In addition there was provided a longitudinal member for taking up the drag. The amplitudes at the forward plane of the spring were read on a gage. The side walls were entirely independent of the test set-up. In order to test the effect of the wing without walls it was made possible to add to the wing end section a rounded piece whose weight and radius of gyration were otherwise replaced in the wing by added weights on the wing. The elastic radius was varied by cutting down the springs, and the mass and the center of gravity as well as the radius of gyration by adding weights to the wing.

In all cases, it was found that the removal of the walls did not result in an increase in the critical velocity but occasionally even in a small decrease (table I). From this result it is to be concluded that no reduction factor need be applied for the tip flow of the three-dimensional wing. The lowering of the critical velocity on removal of the walls is to be explained by the fact that in the case of the short and light wing the boundary layer between the wing and wall disturbs the process and thereby appreciably raises the critical velocity. The critical velocities obtained without the walls are thus more reliable than those obtained with the walls. A rounding off of the wing-tip section had no appreciable effect on the final result.

On table I are also presented the theoretically computed critical velocities. The deviations, which are at times quite considerable, between theory and experiment are probably to be ascribed to the causes discussed above of turbulence and vibration of the jet. Bearing in mind, however, that the wing is so sensitive to even the smallest disturbances the agreement in the two- and three-dimensional cases is surprisingly good.

## IX. FURTHER TESTS PLANNED IN TEST PROGRAM

As a continuation of the above tests the following further investigations are planned:

1. Short supplementary tests on a particularly light wing in the moderate-sized wind tunnel.

2. Investigation of a symmetrical tapered wing of aspect ratio 1:6 in the large wind tunnel to check the assumptions on the three-dimensional problem and to investigate the coupling between the torsional and flexural vibrations.

3. Further investigations with ailerons (ref. 4) for checking the theory with three degrees of freedom and particularly for explaining the effect of the aileron slot on the oscillation process, an effect which is probably quite considerable and which so far has not been taken into account by any theory. It is planned to have the models so constructed as to make it possible to test individual industrial designs by means of models in the wind tunnel within the shortest time possible and thus effect a time-saving in making the extremely extensive computations required for the case of the wing with aileron (three degrees of freedom).

## X. SUMMARY

It is shown that the critical velocity obtained experimentally on a model wing suspended between two walls agreed with the value theoretically computed. For a wing of extremely low aspect ratio the removal of the end walls does not raise the critical velocity. It is thus permissible to apply the computations of the two-dimensional theory without a correction factor to the three-dimensional case.

In the case of extremely light wings of large chord, no absolute, complete agreement between theory and experiment could be obtained on account of the mutual interference effect between tunnel and wing. Further tests in this direction in another tunnel and additional investigations on wings with ailerons are planned.

Translation by S. Reiss,
National Advisory Committee for Aeronautics.

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TABLE I:  Compression Between Two- and Three-Dimensional Wing  a) Wing between End Walls b) Wing without Walls  c) Wing without Walls and Rounding of Tips.  Chord, 500 mm; Span, 300 mm;  Flexural Stiffness 0.68 g/cm;  Elastic Axis at 0.324 Chord

| Wing data | | | | $v_k$ m/s computation | | $v_k$ m/s test | | | Type of oscillation | | | Amplitudes mm | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $\eta^2$ | $\mu$ | $\epsilon + \sigma$ | $i^2$ | Wagner-Kassner | Küssner | Condition a | b | c | a | b | c | a | b | c |
| 0.042 | 2.16 | 0.133 | 0.091 | 3.9 | - | 23 | 23.5 | 23.5 | < > | < > | < > | 10 | 8-20 | 10-20 |
| .042 | 2.66 | .174 | .081 | 3.2 | - | 12.3 | - | 11.3 | = | / | = | 15 | -- | 18 |
| .042 | 3.16 | .202 | .073 | 1.5 | -- | 11.8 | 11.8 | 11.6 | = | = | = | 10 | 25 | 50 |
| .090 | 2.40 | .155 | .092 | $\infty$ | $\infty$ | 18.0 | 15.6 | 15.6 | < > | < > | < > | 0-25 | 8-15 | 0-75 |
| .090 | 2.66 | .173 | .081 | 14.8 | 15.1 | 13.7 | -- | 14.9 | = | / | = | 20 | -- | 25 |
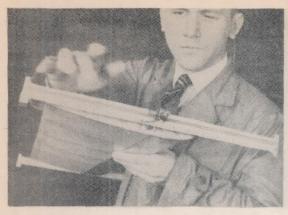
Figure 1.- Wing with side attachement parts.



Figure 2.- Side attachment with added weight and mirror mounting.



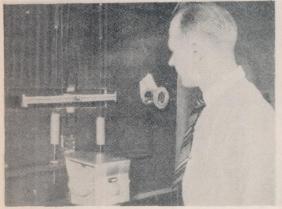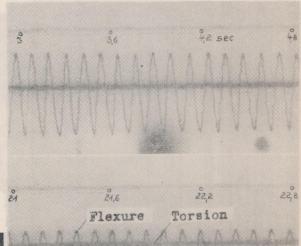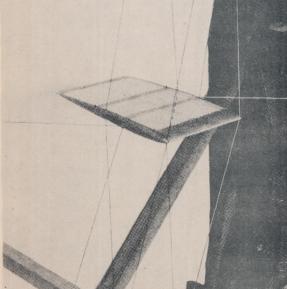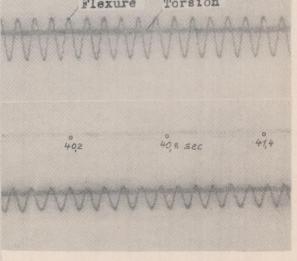Figure 3.- Suspension of wing between end walls.



Flexure    Torsion

Figure 4.- Portion of oscillation ( flexural) curve for determing the natural damping of the system. Wing replaced by tube. Oscillating mass $1/g.240$ g corresponding to $\mu = 4.8 \frac{\theta}{\pi}$ 0.0012, $\nu = 55$ per sec.



Figure 15.- Tests for three dimensional problem. Photo shows wing employed without side walls and with rounded edge pieces.
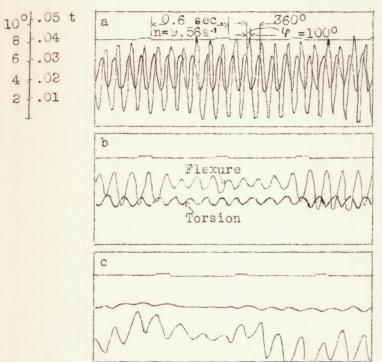
Figure 5.- Examples of observed oscilliation curves.
a) undamped, uniform oscillation phase angle 100° (denoted by sign = in figs. 8 to 13).
b) Oscillation disturbed by irregularities in jet ( denoted by ( < > ).
c) Flexural-torsional oscilliation near the limit of static stability. Slow travel of the wing upwards.
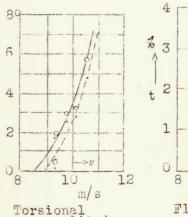


Torsional amplitudes.

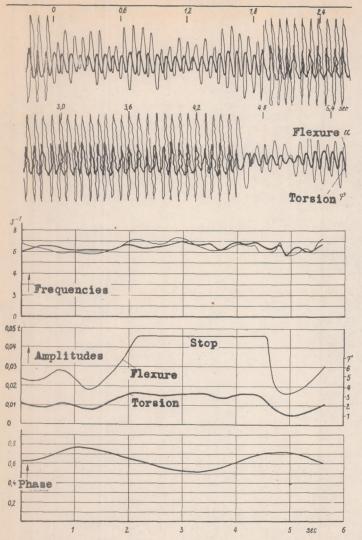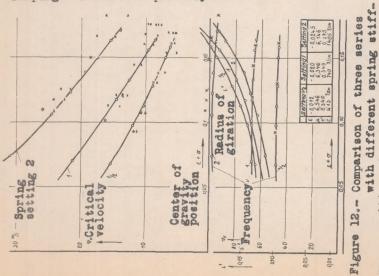Flexural amplitudes.

Figure 6.- Increase in oscillation amplitude with velocity.

Figure 7.- Oscilliation record with determination
of frequencies, amplitudes and phase.
Damping for the most part dry friction.
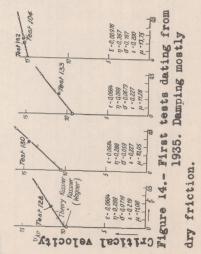


Figure 13.- Effect of damping
in theory and
experiment. Data according to
fig. 8.



Figure 14.- First tests dating from
1935. Damping mostly
dry friction.



Figure 12.- Comparison of three series
with different spring stiff-
ness, position of center of gravity
varied (series 350).

Figure 8.- Comparison between computation and test. Position of center of gravity varied (series 13).

Figure 9.- Comparison between computation and test. Elastic radius varied. Radii of gyration small (series 21/25/2).

Figure 10.- Radii of gyration greater than in fig. 9 (series 21/25/4).

Figure 11.- Radii of gyration greater than in fig. 9 and 10 (21/25/6).